a. $\sin^3 x + \sin^3 |x|$ when $x > 0$

$|x| = x$

$\sin^3 x + \sin^3 \cancel{x}$

$\frac{d}{dx} 2\sin^3 x = 3(2)\sin^2 x \cdot \frac{d}{dx}(\sin)x$

$\frac{dy}{dx} = 6\sin^2 x(\cos x)$

(b) when $x < 0$; $|x| \neq x$ $|x| = (-1)x$

$f(x) = x^3$; x^3

$x \to |x| \to \sin(\ ) \to (\ )^3$

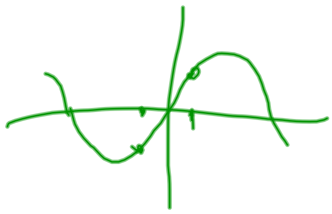
$g(x) = \sin(x)$; $\sin|x|$

$f'(x) = (3(\sin x)^2 \cdot \cos x) + (3(\sin|x|)^2 \cdot \cos|x|)$

$f'(x) = (3\sin^2 x \cos x + 3\sin^2|x| \cos|x| \cdot \frac{d}{dx}(|x|))$

$f(x) = \sin^3 x + \sin^3(|x|)$

$= \sin^3 x + \sin^3(-x)$



$\sin(-x) = -\sin(x)$

sine is an odd f^n

$\sin^3 x + (-\sin(x))^3$

$= \sin^3 x - \sin^3 x$

$= 0$

$\frac{d}{dx}(0) = 0$

$\frac{dy}{dx} =$

$3\sin^2(x) \cdot \cos(x) + 3\sin^2(-x) \cdot \cos(-x) \frac{d}{dx}(-$

$= 3\sin^2 x \cos x - 3\sin^2(-x)\cos(-x)$

c) $f(x)$ cont @ $x=0$?

$$\lim_{x\to 0}\left(\sin^3 x+\sin^3|x|\right) = \sin^3 0+\sin^3|0| = 0$$

$$= \lim_{x\to 0}\left(\sin^3(0)+\sin^3(0)\right)$$

$$= 0$$

yes, continuous b/c the point (0,0) exists, the 2 sided limit exists, & they meet at the point!

$$\lim_{x\to 0} \stackrel{?}{=} 0$$

$$\lim_{x\to 0^-} \frac{d}{dx} 0$$

$$= \lim_{x\to 0^-} 0$$

$$\lim_{x\to 0^+} 6\sin^2 x\cos x$$

$$= \lim_{x\to 0^+} 6\sin^2(0)\cos(0)$$

$$= \lim_{x\to 0^+} 6(0)^2(1)$$

$$= \lim_{x\to 0^+} 0$$

$=$

Yes, $\lim f'(x)$ is cont. @ $x=0$ b/c pt. $(0,0)$ is defined, exists @ 0, and both one-sided limits are the same value as the limit of the derivative approaches 0.

$$f'(0) = \lim_{w \to 0} \frac{f(w) - f(0)}{w - 0}$$

$$= \lim_{w \to 0} \frac{\sin^3(w) + \sin^3(|w|) - 0}{w}$$

$$\lim_{w \to 0^+} \frac{\sin^3 w + \sin^3 w}{w}$$

$$= \lim_{w \to 0^+} \frac{2\sin^3 w}{w}$$

$$= \lim_{w \to 0^+} \underbrace{(2\sin^2 w)}_{0} \underbrace{\left(\frac{\sin w}{w}\right)}_{1}$$

$$= (2)(0)(1)$$

$$= 0$$

$$\lim_{w \to 0^-} \frac{\sin^3 w + \sin^3(|w|) - 0}{w - 0}$$

$$= \lim_{w \to 0^-} \frac{0 - 0}{w - 0} = 0$$

$$\lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1$$

$$\therefore f'(0) = 0$$

3.6/16

$$x^2 = \frac{x+y}{x-y}$$

$$\frac{d}{dx}(x^2) = \frac{d}{dx}\left(\frac{x+y}{x-y}\right)$$

$$2x = \frac{\overset{f'}{\left(1+\frac{dy}{dx}\right)}\overset{g}{(x-y)} - \overset{f}{(x+y)}\overset{g'}{\left(1-\frac{dy}{dx}\right)}}{(x-y)^2}$$

$$2x(x-y)^2 = \left(1+\frac{dy}{dx}\right)(x-y) - (x+y)\left(1-\frac{dy}{dx}\right)$$

$$2x(x-y)^2 = x-y+x\frac{dy}{dx}-y\frac{dy}{dx}-x-y+x\frac{dy}{dx}+y\frac{dy}{dx}$$

$$2x(x-y)^2 = -2y+2x\frac{dy}{dx}$$

$$2x(x-y)^2+2y = 2x\frac{dy}{dx}$$

$$\left(\frac{2x(x-y)^2+2y}{2x}\right) = \frac{dy}{dx}$$

$$\frac{2x(x-y)^2}{2x} + \frac{2y}{2x} = \frac{dy}{dx}$$

$$(x-y)^2 + \frac{y}{x} = \frac{dy}{dx}$$

$3.5/47 \quad y = \tan(4x^2); \quad x = \sqrt{\pi}$

write me a tan line eqⁿ

slope

$f'(x) = \sec^2(4x^2) \cdot 8x$

Value of $f'(x)$ at $x = \sqrt{\pi}$ = slope

$\text{slope} = 8(\sqrt{\pi})\sec^2(4\pi)$

$= 8\sqrt{\pi}$

a pt

$x = \sqrt{\pi}$

$y = \tan(4(\sqrt{\pi})^2)$

$= \tan(4\pi) =$

$\frac{\sin(4\pi)}{\cos(4\pi)} = \frac{\sin(0+4\pi)}{\cos(0+4\pi)}$

$= \frac{0}{1} = 0$

eqⁿ of TL: $y - (0) = 8\sqrt{\pi}(x - \sqrt{\pi})$

3.6/5

$$y = x^3(5x^2+1)^{-2/3}$$

$$\frac{dy}{dx} = (3x^2)\left((5x^2+1)^{-\frac{2}{3}}\right) + (x^3)\left(\frac{d}{dx}(5x^2+1)^{-\frac{2}{3}}\right)$$

$$\frac{dy}{dx} = 3x^2(5x^2+1)^{-2/3} + (x^3)\left(-\frac{2}{3}(5x^2+1)^{-\frac{5}{3}}(10x)\right)$$

$$\frac{d}{dx}(x) = \frac{dx}{dx} = 1$$

$$f(x) = (\sin(x))^3 + (\sin(|x|))^3 \qquad |x| = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$$

$$(\sin(x))^3 + (\sin(x))^3$$

$$2(\sin(x))^3$$

$$\frac{d}{dx}\left(2(\sin(x))^3\right)$$

$$2\frac{d}{dx}(\sin(x))^3$$

$$2\left(3(\sin(x))^2\cos(x)\right)$$

when

$x < 0 \quad \sin^3(x) + \sin^3(-x)$

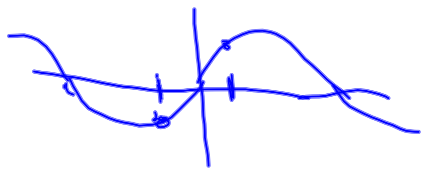
$|x| = -x \quad \sin^3(x) - \sin^3(x)$

$= 0$

$\frac{d}{dx}(0) = 0$

$\sin(-x) = -\sin x$

sine is an ODD f^n



$$\frac{d}{dx}\left(\sin^3(x) + \sin^3(-x)\right)$$

$$3\sin^2(x)\cos(x) + 3\sin^2(-x)\cos(-x)(-1)$$

c) 3 conditions = Defined at c ✓

2 sided limit ✓

2 sided limit = Value of function ✓

$f(x) = \sin^3(x) + \sin^3(|x|)$

$f(0) = \sin^3(0) + \sin^3(|0|)$

$0 + 0 = 0$

$$\lim_{x \to 0} \sin^3(x) + \sin^3(|x|) = 0$$

$f(0) = 0 = 0 = \lim_{x \to 0} f(x)$ $\therefore$ $f(x)$ is cont. at $x = 0$

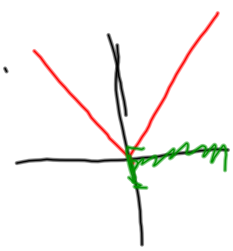
$$f(x) = \begin{cases} \sin^3 x + \sin^3 x = 2\sin^3 x & ; x > 0 \\ 0 & ; x \leq 0 \end{cases}$$

$$f'(x) = \begin{cases} 6\sin^2 x \cos x & ; x > 0 \\ 0 & ; x < 0 \\ 0 & ; x = 0 \end{cases}$$

$$\frac{d}{dx}\left(|x|\right) \qquad |x| = \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases}$$

$$\frac{d}{dx}\left(|x|\right) = \begin{cases} 1 & ; x > 0 \\ -1 & ; x < 0 \\ \text{dne} & ; x = 0 \end{cases}$$

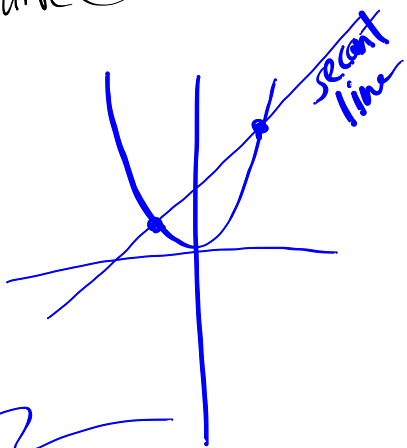
A derivative is a 2 sided limit.



3.3/59) Find x-coord of the pt on graph of $y = x^2$ where the tangent line is $\parallel$ to secant line that cuts the curve @ $x = -1$ and $x = 2$.

slope

f'



$$m_{\text{secant line}} = \frac{f(2) - f(-1)}{2-(-1)}$$

$$= \frac{4-(1)}{3} = 1$$

slope of tangent lines is given by VALUE of the derivative

* find $f'(x) = 2x$

* set $\begin{pmatrix}\text{deriv}\\ \text{fn}\end{pmatrix} = \begin{pmatrix}m \text{ secant}\\ \text{line}\end{pmatrix}$

$$2x = 1$$

$$x = \frac{1}{2}$$

& this is the pt at which ....

slope AT $x = \frac{1}{2}$: $2\left(\frac{1}{2}\right) = 1$

3.6/16)

imp.diff

$$x^2 = \frac{x+y}{x-y}$$

$$\frac{x+y}{x-y} = \frac{x-y+2y}{x-y} = 1 + \frac{2y}{(x-y)}$$

$$\frac{d}{dx}\left(x^2\right) = \frac{d}{dx}\left(\frac{x+y}{x-y}\right)$$

$$2x = \frac{\left(1+\frac{dy}{dx}\right)(x-y) - (x+y)\left(1-\frac{dy}{dx}\right)}{(x-y)^2}$$

$$2x(x-y)^2 = \left(1+\frac{dy}{dx}\right)(x-y) - (x+y)\left(1-\frac{dy}{dx}\right)$$

$$2x(x-y)^2 = (x-y) + x\frac{dy}{dx} - y\frac{dy}{dx} - (x+y) + x\frac{dy}{dx} + y\frac{dy}{dx}$$

$$2x(x-y)^2 = -2y + 2x\frac{dy}{dx}$$

$$2x(x-y)^2 + 2y = 2x\frac{dy}{dx}$$

$$\frac{2x(x-y)^2 + 2y}{2x} = \frac{dy}{dx}$$

$$\frac{\boxed{2x}(x-y)^2}{\boxed{2x}} + \frac{2y}{2x} = \frac{dy}{dx}$$

$$\frac{2y}{2x} = \left(\frac{2}{2}\right)\left(\frac{y}{x}\right) = \frac{y}{x}$$

$$(x-y)^2 + \frac{y}{x} = \frac{dy}{dx}$$

3.6/21 $3x^2-4y^2=7$ find $\frac{d^2y}{dx^2}$ by I.D

Deriv 1

$$6x-8y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{3x}{4y}$$

Deriv 2

$$\frac{d^2y}{dx^2}=\frac{(3)(4y)-(3x)(4\frac{dy}{dx})}{(4y)^2}$$

$$\frac{d^2y}{dx^2}=\frac{12y-12x\left(\frac{3x}{4y}\right)}{16y^2}$$

$$=\frac{12y-\frac{9x^2}{y}}{16y^2}=\frac{\frac{12y^2-9x^2}{y}}{16y^2}$$

$$=\frac{12y^2-9x^2}{16y^3}$$